# Mastering Mental Math: Capture the Power

**Three Methods for Teaching Mental Math to Middle & High School Students**

## Introduction

Mastering Mental Math will enable your students to capture the power they have – and probably don't even *know* they have – right between their ears.

Learning in general is like exploring an endless castle full of different wings and rooms and corridors. Mental math is the light switch that illuminates a new, great hall in that castle, or the flashlight that illuminates other, fascinating parts of the castle. Mental math is empowering.

Mental math is really just arithmetic, and arithmetic – both basic facts and more advanced operations – can and should be done mentally. But before we dive in, let's explain the details.

This resource is not designed to teach basic math facts. Math facts are the core building blocks of mental math. Students who do not have basic facts memorized are like toddlers who struggle to piece together two blocks, let alone build a tower or a pyramid. Like a pyramid, math is a subject that builds on itself, piecing together prerequisite concepts to form more advanced ideas.

This resource is designed to take students with a working fluency of basic facts to the next level. (See sidebar for *What are Basic Math Facts?*) What does that look like? Imagine what a student would do if asked to add 67 and 34, or to multiply 85 and 7. Most will reach for their calculator. After all, these are not basic math facts! This resource provides step-by-step illustrations to teach a student how these problems can be done mentally.

### What are Basic Math Facts?

Adding, Subtracting, Multiplying & Dividing numbers 0 to ±10 and their fact families, like $7, 8$ and $15$, *or* $6, 9$ and $54$.

*arithmetic*

*n. uh*-**rith**-m*uh*-tik

the branch of mathematics concerned with numerical calculations, such as addition, subtraction, multiplication, and division

arithmetic. (n.d.). *Dictionary.com Unabridged*. Retrieved August 03, 2015, from Dictionary.com website: http://dictionary.reference.com/browse/arithmetic

Some students who do not have a certain level of basic math facts will need remediation, and should return to this resource after receiving that extra help. Resources for remediation are readily available online. That said, contact me and I will send you some simple yet rich materials for building up basic math facts mastery. The resource you are viewing here is best utilized when a student has a working fluency of basic math facts.

# Methods

In **Mastering Mental Math: Capture the Power** we will use the following 3 methods:

I. Estimating
II. Mental Computing
III. Using Algebra

**<u>Estimating</u>** is crucial because this skill is used every day by everyone in all walks of life. Whether it is approximating change back or a shopping discount, the skill of estimation is everywhere.

While all three of these methods fall under mental math, the **<u>Mental Computing</u>** method is usually the best way to make a simple calculation.

Sometimes, however, **<u>Using Algebra</u>** may prove helpful. Sometimes it is referred to as conservation, but that sounds too abstract for most teenagers. We're taking some freedom here but the idea underlying algebra is the notion of the *unknown* or hidden number. We make a few comments that call out that "hidden number" in the examples below. Also, calling this method "Using Algebra" will boost the value and accessibility of algebra in the minds of your students, as they push into the more advanced topics beyond arithmetic.

With certain computations it is more efficient to mentally compute using algebra. For example, the last example on page 7, summing 98 and 17 would be best calculated mentally as

$$(98 + 2) + (17 - 2) = 100 + 15 = 115 \quad \text{rather than}$$

$$(90 + 8) + (10 + 7) = (90 + 10) + (8 + 7) = 100 + 15 = 115$$

# Standards Alignment

**Common Core State Standards for Mathematics**

Mental math is a thread that runs through the Common Core standards from elementary to high school. Fluent computation (i.e., from memory) with one-digit numbers appears in the Common Core State Standards for Mathematics as early as Grade 1 (Operations and Algebraic Thinking 1.OA, 6. *Add and subtract within 20*). Standards for Grades 3-5 include *assessing the reasonableness of answers using mental computation* (Operations and Algebraic Thinking 3.OA.8, 4.OA.3, and 7.EE.3.

These standards for computational fluency, or mental math, are critical, even foundational, to the middle and secondary grades' standards. Without such fluency the following standards are, at best, a struggle to meet, and at worst impossible to achieve:

- ratios and proportions (6.RP.2)
- fractions (6.NS.1)
- decimals (6.NS.2-4)
- rational numbers (6.NS.1-8)
- expressions and equations (all standards rely on fluency, but 6.EE.1-5 *Apply and extend previous understandings of arithmetic to algebraic expressions* is especially definitive

This short list includes just a sampling from the 6th (!) Grade Standards. We can safely conclude that the other middle school grades as well as the high school courses demand the same computational fluency for success.

**Principles & Standards for School Mathematics*, *National Council of Teachers of Mathematics**

*Number and Operation Standard*

Instructional programs from prekindergarten through grade 12 should enable all students to

1) Understand numbers, ways of representing numbers, relationships among numbers, and number systems
2) Understand meanings of operations and how they relate to one another
3) Compute fluently and make reasonable estimates

Specific expectations for Grades 9-12 are that students should (a) judge the effects of such operations as *multiplication*, *division*, and computing powers and roots on the magnitudes of quantities, and (b) *develop fluency in operations with real numbers*, … *using mental computation* (emphasis mine).

**Response to Standards**

It is our contention that mental math is the basis for the ability to judge the effect of various operations on magnitudes of quantities, and that fluency in operations can be measured by mental acuity in making these computations. Also, another expectation – for students in grades 3-5 (!) – is that all students should

> develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems , like 30 x 50 (NCTM, 2000).

The illustrations found below are really designed to help teachers and tutors of older middle and high school students who failed to master this elementary-level expectation. Therefore, the illustrations are intentionally not designed for elementary-aged students, although advanced elementary students could benefit from learning the strategies taught. My rising third-grader has. ☺

The illustration pages are also not designed with the student in mind, but the tutor, teacher, or homeschooling parent. The illustration pages can certainly be shared with students but this resource is designed with a more individualized method in mind.

That method could be utilized face-to-face in a 1-on-1 tutoring session, or in a classroom using a conversational teaching style, or in a homeschooling environment with any number of students.

Please contact me with any questions or ideas to share.

All the best to you in your educational endeavors.

References

National Governors Association Center for Best Practices, Council of Chief State School Officers. *Common Core State Standards*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010.

National Council of Teachers of Mathematics. *Principles & Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 2000.

**Mark B. Anderson**

Teacher, tutor and Founder/CEO of Strength in Numbers Tutoring, Inc.

Contact me via LinkedIn, Facebook or at www.strengthinnumberstutoring.org for information on teaching, tutoring and training resources, content development, and workshops for teachers, tutors, and parents.

# Addition

## Example 1

### Estimating

| | | | |
|---|---|---|---|
| 57 | → | 60 | * rounded up |
| + 19 | → | + 20 | * rounded up |
| | | 80 | * estimate is greater than actual |

### Mental Computing

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 57 | = | 50 | + | 7 | | | |
| + 19 | = | + 10 | + | 9 | | | |
| | | 60 | + | 16 | = | 76 | * yes, our estimate was high |

Or just rewrite under the original

57
+ 19
16
+ 60
76

The method shown here – and not the "carry over" method many of us learned as children – reflects how our brains our naturally think. As teachers and tutors we can show this method (perhaps using smaller numbers like 12 + 9) tangibly with blocks or whatever small objects may be available. This small step will help get our older students get comfortable breaking apart numbers and regrouping them in a way that helps add them faster.

### Using Algebra

$57 + 19$

$= 57 + (20 - 1)$

$= (57 + 20) - 1$

$= 77 - 1$

$= 76$

Many older students will be able to add 20 to 57 without much difficulty. But this problem may cause more anxiety especially if you're asking for the answer without use of pencil and paper. There are 2 key questions to help the student make this jump:

1) *19 is more or less than 20?* *[less]*

*2) How much less?* *[one less]*

You may find the student takes the last step independently: *Oh, so it should be one less than 77 !* Talking it through AND showing the work shown here is very powerful. Not only is it powerful but activities like this are also crucial for building up to algebraic thinking.

**Mental Math: Addition Practice #1**

Try all three methods for each problem. And yes, do write out the mental computing. ☺ Once this is learned it will be done mentally and won't be necessary to be done on paper. While the mental computing has typically a single best method, there are ultimately several ways to get an answer with each method. Do you see a pattern?

| | ESTIMATING | MENTAL COMPUTING | USING ALGEBRA |
|---|---|---|---|
| **1)** 28<br>+ 13 | 30<br>+ 10<br>40 | 28 = 20 + 8<br>+ 13 = 10 + 3<br>30 + 11 = 41 | 28 + 2 = 30<br>+ 13 – 2 = 11<br>41 |
| **2)** 38<br>+ 14 | 40<br>+ 10<br>50 | 38 = 30 + 8<br>+ 14 = 10 + 4<br>40 + 12 = 52 | 38 + 2 = 40<br>+ 14 – 2 = 12<br>52 |
| **3)** 48<br>+ 15 | 50 OR 50<br>+ 20 + 15<br>70 65 | 48 = 40 + 8<br>+ 15 = 10 + 5<br>50 + 13 = 63 | 48 + 2 = 50<br>+ 15 – 2 = 13<br>63 |

Fill in the next couple problems yourself and jot down the questions you might ask while walking a student through the examples.

**4)** 68
+ 16

**5)** 99
+ 19

# Subtraction

## Example 1

### Estimating

| | | | |
|---|---|---|---|
| 95 | → | 90 | * rounded down 5 |
| – 32 | → | – 30 | * rounded down 2 |
| | | 60 | * estimate will be less than actual |

### Mental Computing

95 We can subtract straight down since we don't have to borrow.

– 32 Think about the 10's and the 1's separately: 9 tens – 3 tens = 6 tens

63 and 5 ones – 2 ones = 3 ones

### Using Algebra

$95 - 32$

$= 90 + 5 - (30 + 2)$ * Just breaking up each number into 10's and 1's

$= 90 - 30 + 5 - 2$ * Combine the pieces that are alike. Remember we are subtracting *everything* in the () including the 2.

$= 60 + 3$

**Mental Math: Subtraction Practice #1**

Try all three methods for each problem. And yes, do write out the mental computing. ☺ Once this is learned it will be done mentally and won't be necessary to be done on paper. While the mental computing has typically a single best method, there are ultimately several ways to get an answer with each method. Look for patterns! Where there is an * you may want to pose the Question(s) shown to your student.

| | ESTIMATING | MENTAL COMPUTING | USING ALGEBRA |
|---|---|---|---|
| **1)** 93 | 90 | 93 | 90 + 3 |
| – 81 | – 80 | – 81 | – (80 + 1) |
| | 10 | 12 | 10 + 2 = 12 |

| | ESTIMATING | MENTAL COMPUTING | USING ALGEBRA |
|---|---|---|---|
| **2)** 93 | 90 | 93 | 90 + 3 |
| – 80 * | – 80 | – 80 | – (80 + 0) |
| | 10 | 13 | 10 + 3 = 13 |

Q: While our top number is the same, we're subtracting less this time … 80, vs. 81 in #1.
So should our answer (the "difference") be more or less than the answer from #1?

A: More! … OK, let's see work it out and see if we're right.

| | ESTIMATING | MENTAL COMPUTING | USING ALGEBRA |
|---|---|---|---|
| **3)** 93 | 90 | 93 | 90 + 3 = 90 + 3 |
| – 79 *Q1 | – 80 | – 79 | – (80 – 1) = – 80 + 1 *Q2 |
| | 10 | 10 count up… can we add 10? | 10 + 4 = 14 |
| | | + 4 | |
| | | 14 | |

Q1: While our top # is still the same, we're subtracting less this time AGAIN … 79, vs. 80 in #2.
So should our answer (the "difference") be more or less than the answer from #2?

A1: More! … OK, let's see work it out and see if we're right.

Q2: Would you agree that 79 is close to 80?

A2: Yes.

Q2: Let's subtract 80 then. Does that look like a problem we've done before?

A2: Yes, #2.

Q2: Right. How far exactly was 80 from 79?

A2: One.

Q2: So did we subtract too much or too little when we subtracted 80?

A2: Too much… so we should add one back?

Q2: Yes! (If the student has seen the Distributive Property in an Algebra course, you might mention that here, that we essentially distributed the negative sign.)

| | ESTIMATING | MENTAL COMPUTING | USING ALGEBRA |
|---|---|---|---|
| **4)** 93 | 90 | 93 | 90 + 3 = 90 + 3 |
| – 69 * | – 70 | – 69 | – (70 – 1) = – 70 + 1 |
| | 20 | 20 count up… can we add 10? 20? | 20 + 4 = 24 |
| | | + 4 | |
| | | 24 | |

Q: How is this problem like the last one?

A: We're subtracting 10 less. So answer should be 10 more than last time. … YES!